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A TECHNIQUE FOR CONSTRUCTING SPECTRAL REFLECTANCE CURVES FROM VIKING LANDER CAMERA MULTISPECTRAL DATA

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16. Abstract

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SUMMARY

This report presents and evaluates a technique for the construction of spectral reflectance curves from multispectral data obtained with the Viking lander cameras. The multispectral data is limited to 6 channels in the wavelength range 0.4 to 1.1 μm, and several of the channels suffer from appreciable out-of-band response. Previous analysis showed that curve fits through 6 spectral data points plotted at distinct wavelengths often did not reveal important spectral features present in the materials and sometimes generated false spectral features.

The technique proposed herein represents the estimated reflectance curves as a linear combination of known basis functions with coefficients determined to minimize the error in the representation, and it permits all channels, with and without out-of-band response, to contribute equally valid information. The technique is evaluated for known spectral reflectance curves of 8 materials felt likely to be present on the Martian surface. The technique provides an essentially exact fit if the reflectance curve has no pronounced maxima and minima. Even if the curve has pronounced maxima and minima, the fit is good and reveals the most dominant features. Since only 6 samples are available some short period features are lost. This loss is almost certainly due to undersampling rather than out-of-band channel response.

INTRODUCTION

The Viking lander cameras will image the Martian surface with 6 spectral channels in the wavelength range 0.4 to 1.1 μm. It is desired to construct approximate spectral reflectance curves from this data to aid in identifying surface materials. This objective is rendered difficult not only by the small number of channels, but also by the appreciable out-of-band response of several of these channels.

The out-of-band response of spectral channels makes it inappropriate to generate data points by associating spectral reflectance values with distinct wavelengths. That is, it is unlikely that data points so constructed will lie on the true spectral reflectance curve. Consequently, any method of fitting a curve to these data points is quite unlikely to adequately approximate the true spectral reflectance curve. A preliminary evaluation (ref. 1) confirmed this suspicion. It was found that a curve fit through these 6 data points often did not reveal important spectral features and sometimes generated spectral features which were false.

The purpose of this paper is to present and evaluate a more appropriate technique for constructing spectral reflectance curves. In this technique, the curves are represented as a linear combination of known basis functions with the coefficients determined to minimize the error in the representation. Two sets of basis functions are considered which generate, respectively, a 5th degree polynomial and a cubic spline (with 6 knots). The technique is evaluated by using it to approximate the known spectral reflectance curves for 8 materials felt likely to be present on the Martian surface.

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The purpose of this paper is to present and evaluate a more appropriate technique for constructing spectral reflectance curves. In this technique, the curves are represented as a linear combination of known basis functions with the coefficients determined to minimize the error in the representation. Two sets of basis functions are considered which generate, respectively, a 5th degree polynomial and a cubic spline (with 6 knots). The technique is evaluated by using it to approximate the known spectral reflectance curves for 8 materials felt likely to be present on the Martian surface.

FORMULATION

Let $\rho(\lambda)$ be the spectral reflectance of the unknown material that is imaged by the camera. This reflectance can be represented as

$$\rho(\lambda) = \sum_{j=1}^{n} x_j \, h_j(\lambda) + \epsilon(\lambda) \tag{1}$$

where $h_j(\lambda)$ are known basis functions, x_j are (constant) coefficients to be determined, and $\epsilon(\lambda)$ is the error introduced by this representation. As noted in the introduction, two sets of basis functions $h_j(\lambda)$ will be evaluated.

Knowledge of the reflectance $\rho(\lambda)$ is limited to $m(=6)$ spectral samples. Except for a channel dependent, multiplicative constant* these samples are given by m quantities, b_i, where

$$b_i = \int_0^{\infty} \rho(\lambda) T_i(\lambda) \, d\lambda \, , \qquad i = 1,2,\ldots,m \tag{2}$$

The transfer function $T_i(\lambda)$ accounts for solar irradiance and camera responsivity, and is given by

$$T_i(\lambda) = S(\lambda)\tau_a(\lambda)\tau_c(\lambda)R_i(\lambda)$$

where $S(\lambda)$ is the solar irradiance, $\tau_a(\lambda)$ the atmospheric transmittance, $\tau_c(\lambda)$ the camera optical throughput, and $R_i(\lambda)$ the photosensor array responsivity for the i'th channel. Figure 1 presents plots of $T_i(\lambda)$

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* Each of these channel dependent constants are determined by a calibration using as reference a test chart (ref. 1) containing surfaces with known spectral reflectance curves.

for the 6 narrowband channels of the Viking lander camera.

In practice, b_i must be estimated from digital data that is contaminated by electronic and quantization noise as described in reference 2. However, the resulting uncertainties in the actual value of b_i are not considered here.

Substituting equation (1) into (2) yields m equations

$$b_i = \sum_{j=1}^{n} a_{ij} x_j + e_i \qquad i = 1,2,\ldots, m \tag{3}$$

where

$$a_{ij} = \int_0^\infty h_j(\lambda) T_i(\lambda) d\lambda \tag{4}$$

and

$$e_i = \int_0^\infty \epsilon(\lambda) T_i(\lambda) d\lambda \tag{5}$$

The quantities a_{ij} can be calculated once a choice of basis functions $h_j(\lambda)$ has been made. The error terms e_i provide a measure of the difference between the unknown reflectance $\rho(\lambda)$ and the estimate $\sum_{j=1}^{n} x_j h_j(\lambda)$. If $e_i = 0$ for $i = 1,2,\ldots, m$, then, as measured by the 6 spectral channels, the reflectance and the estimate would be indistinguishable (i.e., the reflectance and the estimate would generate identical samples $b_1, b_2, \ldots, b_m$). However, this does not mean that the estimate and the unknown reflectance are necessarily equal for all wavelengths (i.e., that $\epsilon(\lambda)=0$).

In general, the unknown coefficients x_j should be chosen to minimize

the sum of squared errors; that is,

$$e_1^{\,2} + e_2^{\,2} + \ldots + e_m^{\,2} \to \min$$

or equivalently the root-mean-square (rms) error

$$\left(\frac{e_1^{\,2} + e_2^{\,2} + \ldots e_m^{\,2}}{m} \right)^{1/2} \to \min$$

where from equation (3)

$$e_i = b_i - \sum_{j=1}^{n} a_{ij}\, x_j \tag{6}$$

This minimization problem is the well-known (linear) least squares problem, and it can be solved on a digital computer by standard algorithms.

Two special cases might help to illustrate the technique that is presented herein. Let the number of spectral samples be equal to the number of coefficients (i.e., $m = n = 6$). If the basis functions $h_j(\lambda)$ are chosen so that the square matrix A with elements a_{ij} is non-singular (i.e., the inverse, A^{-1}, exists), then $e_i = 0$ for $i = 1,2,\ldots, m$ and the column vector x with components $x_1,x_2,\ldots,x_n$ is uniquely determined by $x = A^{-1}b$ where b is the column vector with components $b_1,b_2,\ldots,b_m$.

That is, in this case the 6 coefficients $x_1,x_2,\ldots,x_6$ are determined by solving 6 (simultaneous) linear equations. Moreover, the reflectance curve and the estimate are indistinguishable.

As a second case consider sampling with spectral filters so narrow that it is appropriate to attach spectral reflectance values to distinct wave-

lengths. That is, the transfer functions $T_i(\lambda)$ can be approximated by impulse functions at distinct wavelengths λ_i as given by

$$T_i(\lambda) = \delta(\lambda-\lambda_i), \qquad i = 1,2,\ldots,m$$

and shown in figure 2(a). Equations (2), (4), and (6) become, respectively,

$$b_i = \int_0^\infty \rho(\lambda)\, \delta(\lambda - \lambda_i)d\lambda = \rho(\lambda_i)$$

$$a_{ij} = \int_0^\infty h_j(\lambda)\, \delta(\lambda - \lambda_i)d\lambda = h_j(\lambda_i)$$

$$e_i = \rho(\lambda_i) - \sum_{j=1}^{n} x_j\, h_j(\lambda_i)$$

The spectral samples b_i and elements a_{ij} are the actual values of the surface reflectance $\rho(\lambda)$ and basis functions $h_j(\lambda)$ respectively at distinct wavelengths λ_i. The coefficients x_j are determined so as to force the estimate $\sum_{j=1}^{n} x_j\, h_j(\lambda)$ to best fit the reflectance at the wavelengths λ_i, as is illustrated in figure 2b. The error term e_i accounts for the amount by which the estimate fails to fit the reflectance at the wavelengths λ_i.

To summarize, the technique proposed herein for constructing spectral reflectance curves from m samples $b_1,b_2,\ldots,b_m$ (eq.(2)) is as follows:

(i) select a set of n basis functions $h_j(\lambda)$ $(n \leq m)$;

(ii) evaluate the mn quantities a_{ij} given by equation (4);

(iii) determine the n coefficients x_j by either solving the linear least squares problem, or, if n=m and the square matrix A with

elements a_{ij} is non-singular, by solving the linear equations (3) with $e_i = 0$;

(iv) form the estimated spectral reflectance $\sum_{j=1}^{n} x_j h_j(\lambda)$.

It should be recognized that this technique permits all channels, with and without out-of-band response, to contribute equally valid information because it does not require spectral reflectance values to be associated with distinct wavelengths.

The relative success (or failure) of the technique is determined by the choice of basis functions. Certainly, some choices are more appropriate than others. In the next section, two sets of basis functions are discussed. Following that, these basis sets are evaluated to demonstrate how accurately they generate approximations to the known spectral reflectance curves of 8 materials.

BASIS FUNCTIONS

If the basis functions are chosen to be $h_1(\lambda) = 1$, $h_2(\lambda) = \lambda, \ldots,$ $h_n(\lambda) = \lambda^{n-1}$ then the estimate $\sum_{j=1}^{n} x_j \lambda^{j-1}$ is an $(n-1)^{th}$ degree polynomial in the variable λ with coefficients x_j. With $n=m=6$ the square matrix A with elements

$$a_{ij} = \int_0^\infty \lambda^{j-1} T_i(\lambda) d\lambda$$

is non-singular so that the polynomial is uniquely determined by solving equations (3) with $e_i = 0$. It is known that high degree polynomial approximations can possess undesirable oscillations. However, the results shown later with n equal to 6 do not exhibit excessive oscillations (except

possibly at the endpoints).

A more modern and usually better method for fitting curves to data involves the use of a cubic spline. A cubic spline is a curve defined on a sequence of subintervals. The end points of these subintervals are known as knots. On each subinterval the curve is a cubic polynomial and at the knots the adjacent polynomials are joined in such a fashion that the resulting curve (the cubic spline) is continuous with continuous 1st and 2nd derivatives.

The number of knots (n) is less than or equal to the number of samples $(m=6)$, and, for convenience, the knots $(\bar{\lambda}_j)$ are chosen to be equally spaced. The knots used for all the calculations discussed herein are

$$\bar{\lambda}_j = .425 + (j-1)\Delta \qquad j = 1,2,\ldots,6$$

where the spacing is $\Delta = .125$ µm. The basis functions are

$$h_j(\lambda) = C(\lambda-\bar{\lambda}_j) \qquad j = 1,2,\ldots,6$$

where $C(\lambda)$ is the function defined by (see figure 3a)

$$C(\lambda) = \begin{cases} \frac{1}{6\Delta^3}\left[\Delta^3 + 3\Delta^2(\Delta-|\lambda|) + 3\Delta(\Delta-|\lambda|)^2 - 3(\Delta-|\lambda|)^3\right] & |\lambda| \le \Delta \\ \frac{1}{6\Delta^3}(2\Delta-|\lambda|)^3 & \Delta < |\lambda| < 2\Delta \\ 0 & |\lambda| \ge 2\Delta \end{cases}$$

On each subinterval (of width Δ) $C(\lambda-\bar{\lambda}_j)$ is a cubic polynomial in λ, and at each knot $C(\lambda-\bar{\lambda}_j)$ is continuous with continuous 1st and 2nd

derivatives. Consequently, the estimate $\sum_{j=1}^{n} x_j \, C(\lambda-\bar{\lambda}_j)$ also has these properties (i.e., it is a cubic spline). The 6 basis functions are indicated in figure (3b).

With $n=m=6$ the matrix A with coefficients

$$a_{ij} = \int_0^\infty C(\lambda-\bar{\lambda}_j) \, T_i(\lambda) d\lambda$$

is non-singular so that the cubic spline $\sum_{j=1}^{n} x_j \, C(\lambda-\bar{\lambda}_j)$ is uniquely determined by equations (3) with $e_i = 0$.

RESULTS

Figure 4 presents the spectral reflectance curves for 8 materials that can be reasonably expected on the surface of Mars. Using the technique discussed previously, 5th degree polynomial and cubic spline estimates were constructed for each material. The results are presented in figures 5 and 6 for the polynomials and cubic splines respectively. In these two figures the estimate is denoted as

$$<\rho(\lambda)> = \sum_{j=1}^{n} x_j \, h_j(\lambda) \qquad (n = 6)$$

and shown as a heavy curve; the true reflectance $\rho(\lambda)$ (from figure 4) is shown as a light curve. The error between real and estimated spectral reflectance

$$\epsilon(\lambda) = \rho(\lambda) - <\rho(\lambda)>$$

is indicated in small inserts.

For computational purposes the 6 transfer functions $T_i(\lambda)$ and 8 spectral reflectance curves $\rho(\lambda)$ were measured at 29 equally spaced wavelengths $\lambda = .4, .425, .450, \ldots, 1.075, 1.1$. The resulting data arrays were used as inputs to determine the samples b_i (eq.(2)) and quantities a_{ij} (eq. (4)) by numerical integration (Simpson's rule).

For both the polynomial and cubic spline estimates there is a tendency for $|\epsilon(\lambda)|$ to become large rapidly as the end points ($\lambda = 0.4$ and $\lambda = 1.1$) are approached. For this reason and due to people's propensity for extrapolation, estimates near the end point must be handled with care. Thus, additional work is needed:

(i) to assert a priori a reduced domain (e.g., $0.5 \leq \lambda \leq 1.0$) in which all estimates are accurate;

(ii) or, to impose natural boundary conditions (e.g., that the 2nd derivative be zero at the end points) in an attempt to avoid having to reduce the domain.

For the 8 materials used herein all estimates are observed to be valid in the reduced domain $.425 \leq \lambda \leq 1.025$. For those materials with relatively simple spectral reflectances (i.e., average Mars, Syrtis Major, pinacates, and augite) the estimates are excellent for both the polynomial and cubic spline.

For those materials with more complex spectral reflectances (i.e., limonite, hypersthene, and olivine) the estimates are very good. The dominant features are reproduced by both estimates; however, short period features are lost. This loss is almost certainly due to the small number of spectral channels rather than the out-of-band response of some channels. For olivine the polynomial and cubic spline estimates are nearly identical, while for

hypersthene and limonite the cubic spline estimate is superior. Specifically note that the cubic spline quite accurately predicts the dominant peak and absorption band for hypersthene. In general the results indicate a preference for the use of cubic splines as the basis functions.

As mentioned previously the analysis herein ignores the fact that each b_i must be estimated from digital data that is contaminated by electronic and quantization noise. Also, the analysis assumes that each channel output can be accurately calibrated by reference to a test chart. Thus, additional work is necessary to determine in practice how accurately spectral reflectance curves can be reconstructed using as inputs samples b_i determined by the actual Viking lander camera channel outputs rather than samples b_i determined theoretically by performing the integration indicated in equation 2.

CONCLUDING REMARKS

A technique has been presented and evaluated for constructing spectral reflectance curves from multispectral data obtained with the Viking lander cameras. The multispectral data is limited to 6 spectral channels in the wavelength range 0.4 to 1.1 μm, and some of the channels suffer from appreciable out-of-band response. The proposed technique presents the estimated reflectance curves as a linear combination of known basis functions with coefficients determined to minimize the error in the representation. An important advantage of this technique is that it permits all channels, with and without out-of-band response, to contribute equally valid information. It properly accounts for the solar irradiance of the surface and the response of the camera without associating multispectral data with distinct wavelengths.

The technique was evaluated for 8 spectral reflectances of materials

that can be reasonably expected on the surface of Mars. Estimated spectral reflectances for materials with relatively simple reflectances resulted in essentially exact fits to the actual curves. Estimated spectral reflectances for materials with relatively complex reflectances still resulted in very good fits; however, short period reflectance features were lost. The loss of short period features is almost certainly the result of undersampling with the limited number of channels rather than the out-of-band response of some of these channels.

REFERENCES

1. Kelly, W. L.; Huck, F. O.; Arvidson, R. E.: Spectral Response of the Viking Lander Camera: Preliminary Evaluation. NASA TM in preparation.

2. Huck, F. O.; Burcher, E. E.; Taylor, E. J.; Wall, S. D.: Radiometric Performance of the Viking Mars Lander Cameras. NASA TM X-72692.

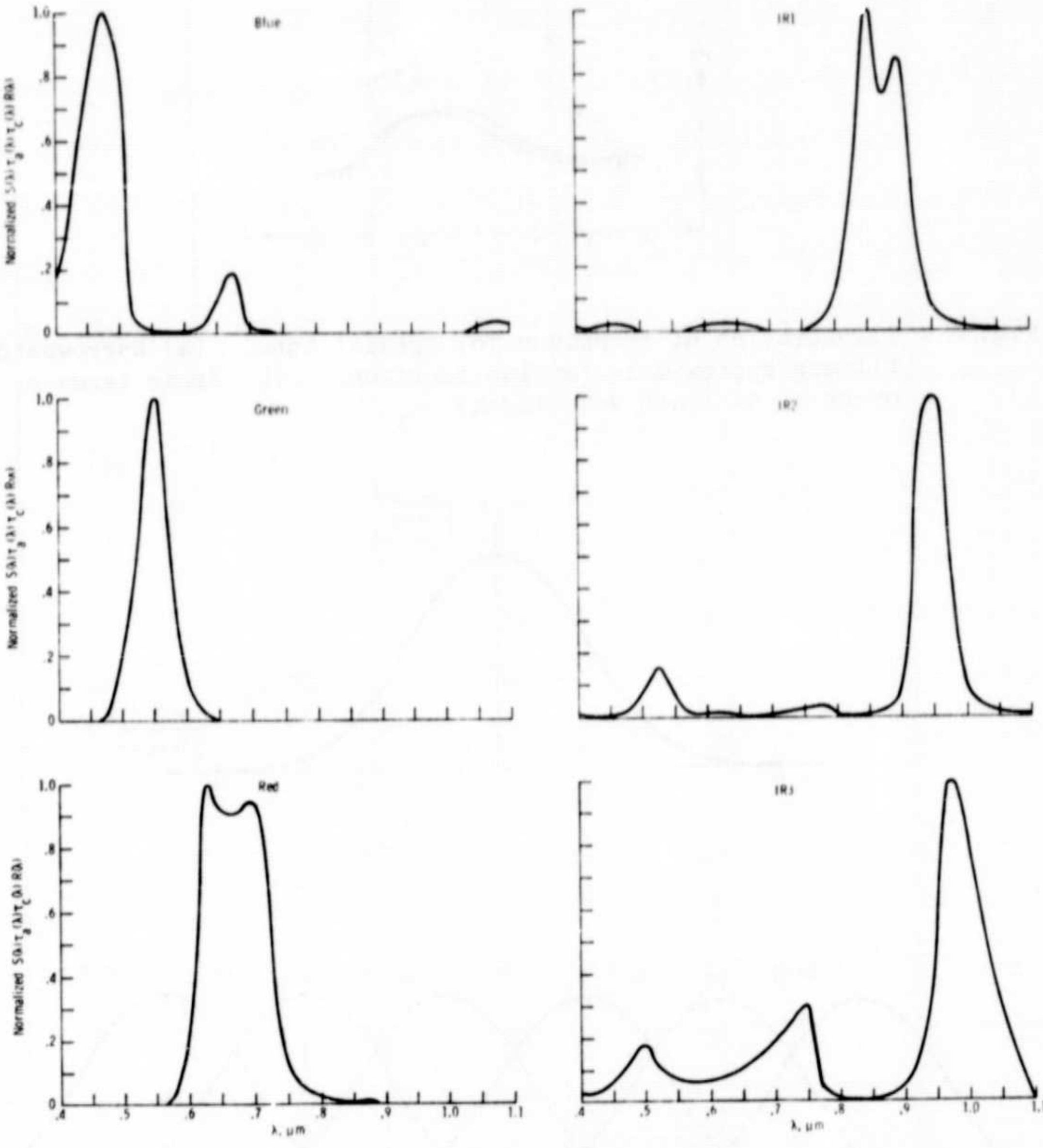

Fig. 1 - Normalized product of solar irradiance, atmospheric transmittance, camera optical throughput, and photosensor array responsivity.

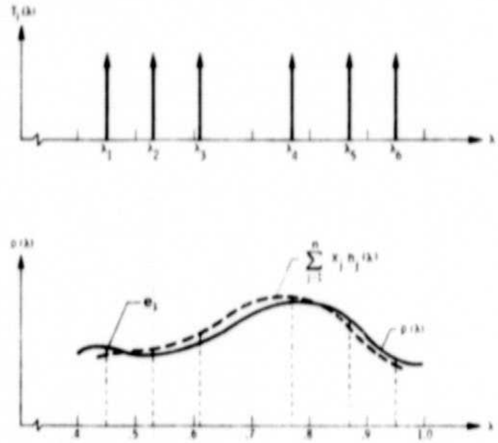

Fig. 2 - Illustration of technique for special case. (a) Narrowband filters approximate impulse functions. (b) Error terms e_i occur at distinct wavelengths λ_i.

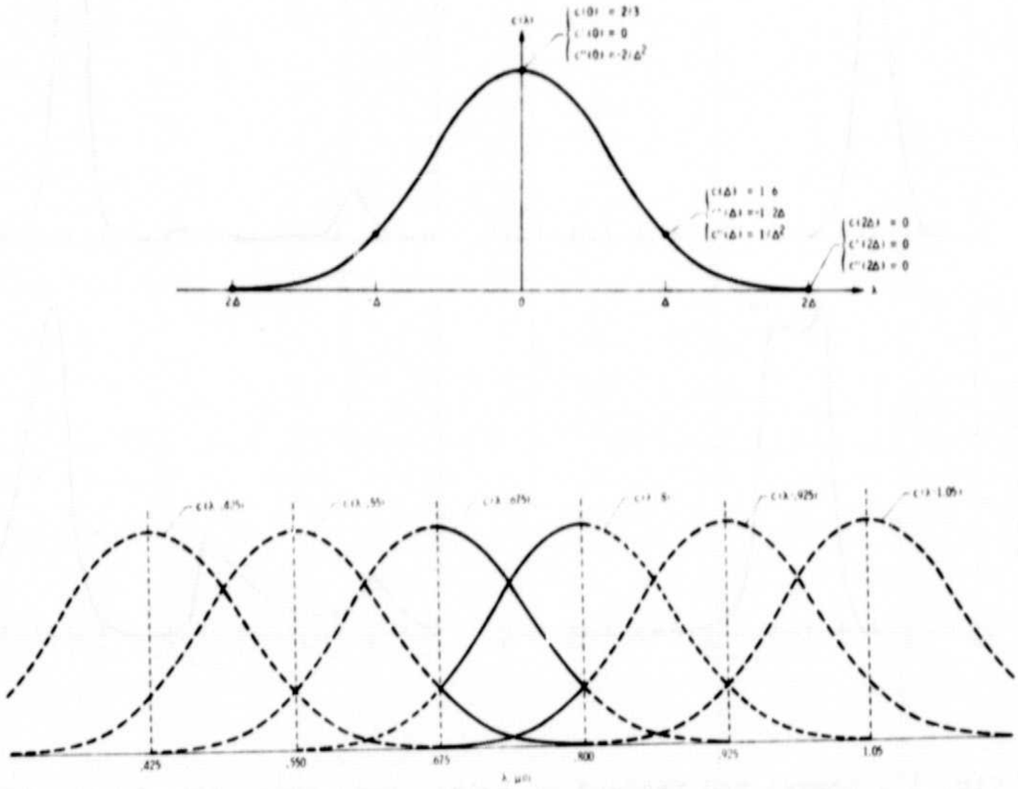

Fig. 3 - Cubic spline. (a) Defining properties. (b) Basis functions used for construction of curves.

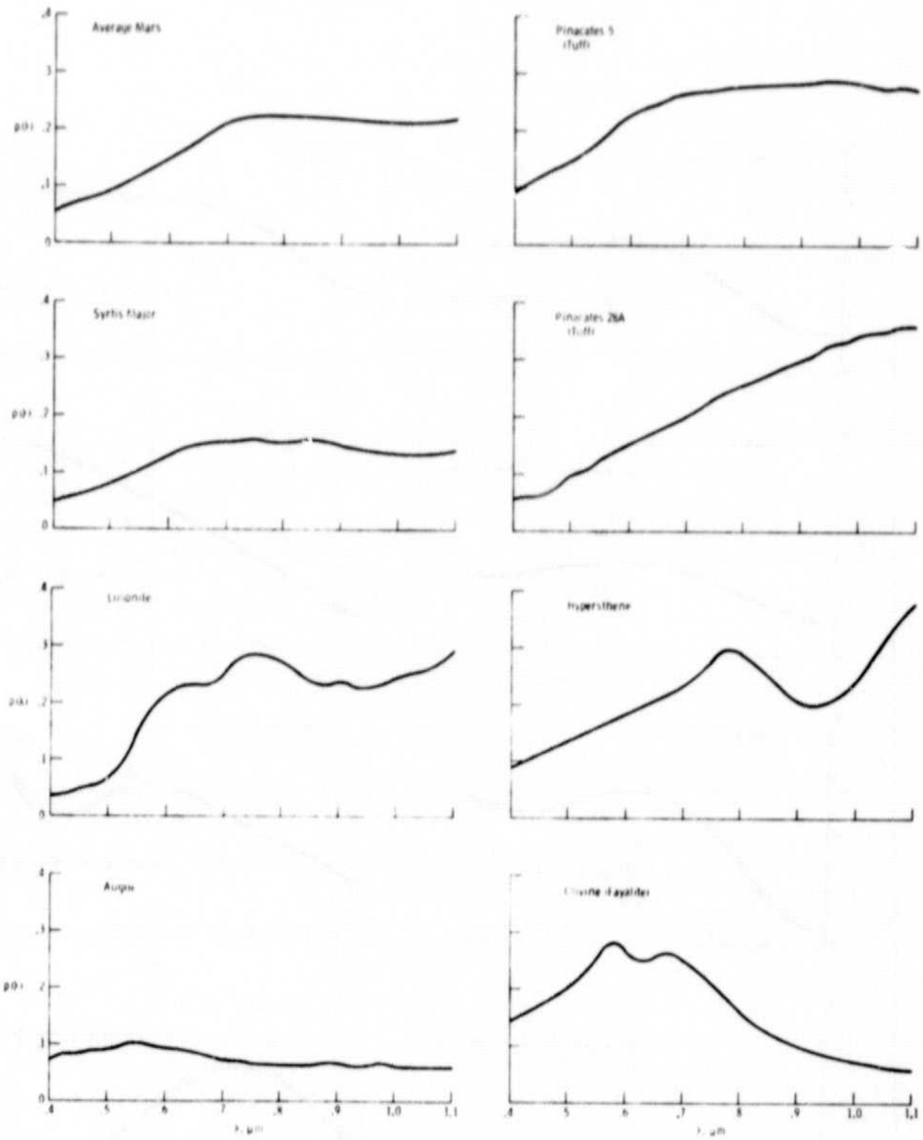

Fig. 4 - Spectral reflectance curves.

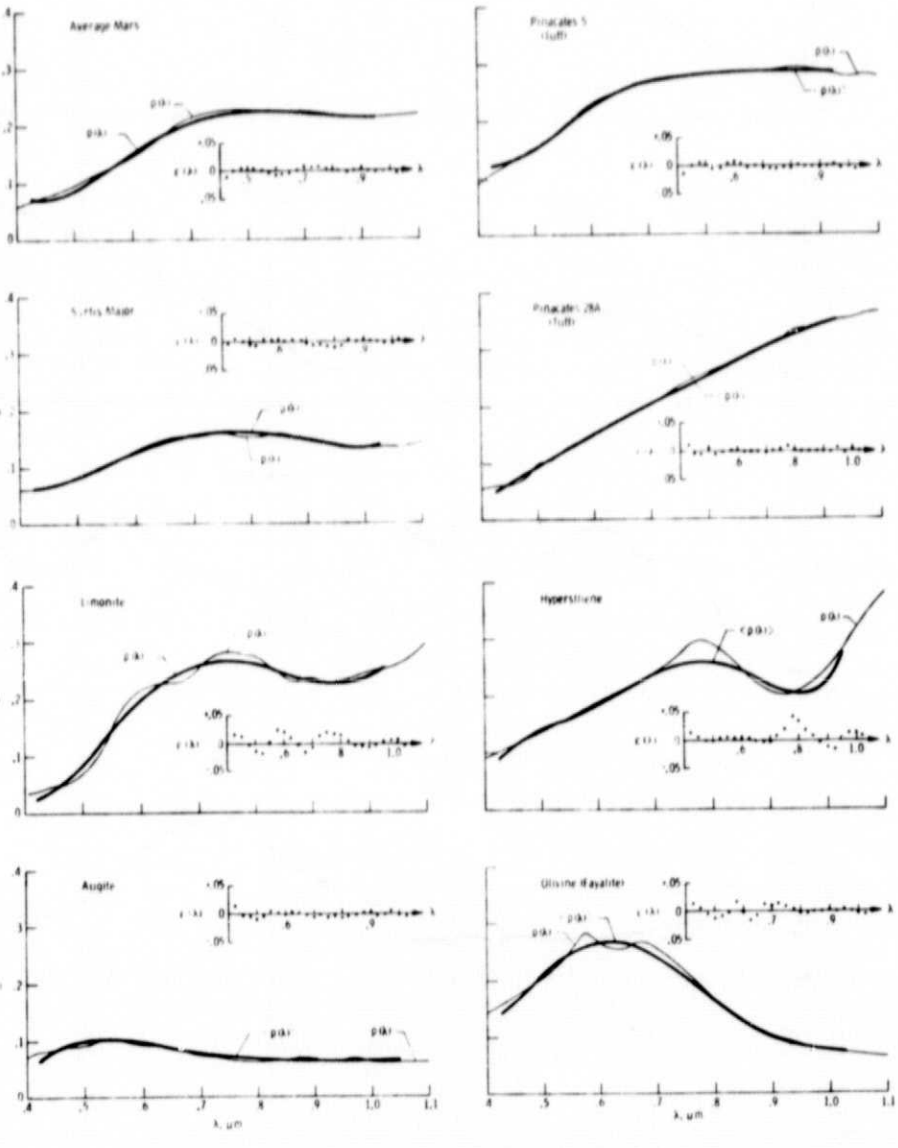

Fig. 5 - Estimated reflectance curves using 5th degree polynomials.

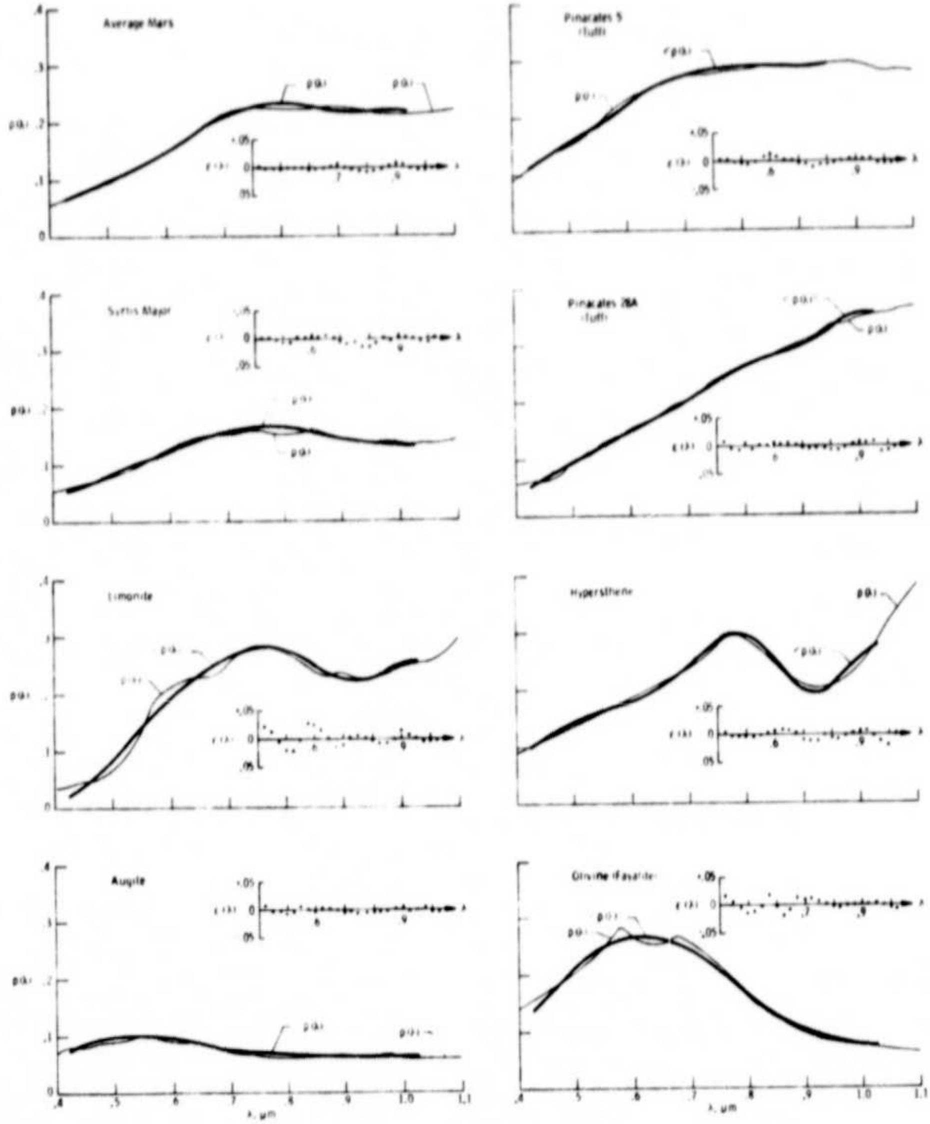

Fig. 6 - Estimated reflectance curves using cubic splines.